For all questions, answer choice "(E) NOTA" means that none of the above answers is correct.

1. **A**) $2A-3B=2\begin{bmatrix}1&2&4\\-3&0&-1\\2&1&2\end{bmatrix}-3\begin{bmatrix}2&2&1\\-1&1&-3\\0&1&2\end{bmatrix}=\begin{bmatrix}2-6&4-6&8-3\\-6+3&0-3&-2+9\\4-0&2-3&4-6\end{bmatrix}=\begin{bmatrix}-4&-2&5\\-3&-3&7\\4&-1&-2\end{bmatrix}$. The correct answer is A.

2. **B**) $3A+4B=3\begin{bmatrix}1&-2&4\\3&0&1\\2&-1&-2\end{bmatrix}+4\begin{bmatrix}2&2&1\\-2&1&-1\\1&-2&2\end{bmatrix}=\begin{bmatrix}3+8&-6+8&12+4\\9-8&0+4&3-4\\6+4&-3-8&-6+8\end{bmatrix}=\begin{bmatrix}11&2&16\\1&4&-1\\10&-11&2\end{bmatrix}$. The correct answer is B.

3. **A**) From given equation, we have $\begin{bmatrix}4x&4y\\4z&-4\end{bmatrix}=\begin{bmatrix}2y+8&2z+2x\\-2x+10&2-2x\end{bmatrix}$. It is easy to see $-4=2-2x\Rightarrow x=3,\ 4z=10-2x\Rightarrow z=1,\ 4x=2y+8\Rightarrow y=2$. Therefore, $x+y-2z=3$.

   The correct answer is A.

4. **D**) It is easy to see

   $AB+B^TA=\begin{bmatrix}1&2\\3&1\end{bmatrix}\begin{bmatrix}4&1\\5&-2\end{bmatrix}+\begin{bmatrix}4&1\\5&-2\end{bmatrix}^T\begin{bmatrix}1&2\\3&1\end{bmatrix}=\begin{bmatrix}1&2\\3&1\end{bmatrix}\begin{bmatrix}4&1\\5&-2\end{bmatrix}+\begin{bmatrix}4&5\\1&-2\end{bmatrix}\begin{bmatrix}1&2\\3&1\end{bmatrix}$

   $=\begin{bmatrix}14&-3\\17&1\end{bmatrix}+\begin{bmatrix}19&13\\-5&0\end{bmatrix}=\begin{bmatrix}33&10\\12&1\end{bmatrix}$.

   The correct answer is D

5. **E**) Solve $\begin{cases}x_1-2x_2+2x_3=7\\x_1-3x_2+4x_3=12\\2x_1-5x_2+3x_3=13\end{cases}$, we have $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\-1\\2\end{bmatrix}\Rightarrow x_1^2+x_2^2+x_3^2=6$. No correct answer provided. Therefore, the correct answer is E.

6. **D**) $\begin{bmatrix}1&2\\1&3\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}a+2c&b+2d\\a+3c&b+3d\end{bmatrix}=\begin{bmatrix}2&1\\4&3\end{bmatrix}\Rightarrow\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}-2&-3\\2&2\end{bmatrix}$, $(a+b+c+d)^2=1$. Therefore, the correct answer is D.

7. **C**) $A^2=\begin{bmatrix}2&1&1\\0&2&1\\0&0&2\end{bmatrix}\begin{bmatrix}2&1&1\\0&2&1\\0&0&2\end{bmatrix}=\begin{bmatrix}2^2&2^2&5\\0&2^2&2^2\\0&0&2^2\end{bmatrix}$ $A^3=\begin{bmatrix}2&1&1\\0&2&1\\0&0&2\end{bmatrix}\begin{bmatrix}2^2&2^2&5\\0&2^2&2^2\\0&0&2^2\end{bmatrix}=\begin{bmatrix}2^3&12&18\\0&2^3&12\\0&0&2^3\end{bmatrix}$

Therefore, the correct answer is C.

8. **B**) $AB=\begin{bmatrix}\cos x&-\sin x\\\sin x&\cos x\end{bmatrix}\begin{bmatrix}\cos y&-\sin y\\\sin y&\cos y\end{bmatrix}=\begin{bmatrix}\cos(x+y)&-\sin(x+y)\\\sin(x+y)&\cos(x+y)\end{bmatrix}$

$BA=\begin{bmatrix}\cos y&-\sin y\\\sin y&\cos y\end{bmatrix}\begin{bmatrix}\cos x&-\sin x\\\sin x&\cos x\end{bmatrix}=\begin{bmatrix}\cos(x+y)&-\sin(x+y)\\\sin(x+y)&\cos(x+y)\end{bmatrix}$ $AB-BA=0$

Therefore, the correct answer is B.

9. **A**) $XA=X\Rightarrow\begin{bmatrix}\frac{1}{2}x_1+\frac{1}{3}x_2+\frac{1}{4}x_3\\\frac{1}{2}x_1+\frac{1}{3}x_2+\frac{1}{2}x_3\\\frac{1}{3}x_2+\frac{1}{4}x_3\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$

$\begin{cases}6x_1+4x_2+3x_3=12x_1\\3x_1+2x_2+3x_3=6x_2\\4x_2+3x_3=12x_3\end{cases}\Rightarrow\begin{cases}4x_2+3x_3=6x_1\\3x_1+3x_3=4x_2\\4x_2=9x_3\end{cases}\Rightarrow\begin{cases}x_1=2x_3\\x_2=\dfrac{9x_3}{4}\\x_3=x_3\end{cases}\Rightarrow\dfrac{x_1}{x_3}=2$

Therefore, the correct answer is A.

10. **C**) $A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}\sim\begin{bmatrix}3&2&3\\6&5&6\\9&8&9\end{bmatrix}\sim\begin{bmatrix}0&2&3\\0&5&6\\0&8&9\end{bmatrix}$ $\text{Rank}(A)=2$

Therefore, the correct answer is C.

11. **A**) $A:I=\begin{bmatrix}1&2&1&1&0&0\\1&1&2&0&1&0\\2&1&1&0&0&1\end{bmatrix}\sim\begin{bmatrix}1&2&1&1&0&0\\0&-1&1&-1&1&0\\0&-3&-1&-2&0&1\end{bmatrix}\sim\begin{bmatrix}1&2&1&1&0&0\\0&-1&1&-1&1&0\\0&0&-4&1&-3&1\end{bmatrix}$

$\sim\begin{bmatrix}1&2&0&-\frac{5}{4}&-\frac{3}{4}&\frac{1}{4}\\0&-1&0&-\frac{3}{4}&\frac{1}{4}&\frac{1}{4}\\0&0&1&-\frac{1}{4}&\frac{3}{4}&-\frac{1}{4}\end{bmatrix}\sim\begin{bmatrix}1&0&0&-\frac{1}{4}&-\frac{1}{4}&\frac{3}{4}\\0&1&0&\frac{3}{4}&-\frac{1}{4}&-\frac{1}{4}\\0&0&1&-\frac{1}{4}&\frac{3}{4}&-\frac{1}{4}\end{bmatrix}\Rightarrow A^{-1}=\begin{bmatrix}-\frac{1}{4}&-\frac{1}{4}&\frac{3}{4}\\\frac{3}{4}&-\frac{1}{4}&-\frac{1}{4}\\-\frac{1}{4}&\frac{3}{4}&-\frac{1}{4}\end{bmatrix}$

$$4A^{-1}=\begin{bmatrix}-\frac{1}{4} & -\frac{1}{4} & \frac{3}{4}\\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4}\\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4}\end{bmatrix}=\begin{bmatrix}-1 & -1 & 3\\ 3 & -1 & -1\\ -1 & 3 & -1\end{bmatrix}$$. Therefore, the correct answer is A.

12. **B**) $A=\begin{bmatrix}1 & 2 & 1\\ 6 & -1 & 0\\ -1 & -2 & -1\end{bmatrix}\Rightarrow \text{cofactors(A)}=\begin{bmatrix}\begin{bmatrix}-1 & 0\\ -2 & -1\end{bmatrix} & -\begin{bmatrix}6 & 0\\ -1 & -1\end{bmatrix} & \begin{bmatrix}6 & -1\\ -1 & -2\end{bmatrix}\\ -\begin{bmatrix}2 & 1\\ -2 & -1\end{bmatrix} & \begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix} & -\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}\\ \begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix} & -\begin{bmatrix}1 & 1\\ 6 & 0\end{bmatrix} & \begin{bmatrix}1 & 2\\ 6 & -1\end{bmatrix}\end{bmatrix}=\begin{bmatrix}1 & 6 & -13\\ 0 & 0 & 0\\ 1 & 6 & -13\end{bmatrix}$

Therefore, the correct answer is B.

13. **C**) $|A-\lambda I|=\begin{vmatrix}1-\lambda & 0 & 2\\ 3 & 1-\lambda & 5\\ 2 & 5 & 8-\lambda\end{vmatrix}=(1-\lambda)^2(8-\lambda)+30-29(1-\lambda)\ \begin{bmatrix}1\\ 1\end{bmatrix}$

$=-\lambda^3+10\lambda^2+12\lambda+9$

Therefore, the correct answer is C.

14. **D**) $\text{tr}(A)=1+3+1+3=8$. Therefore, the correct answer is D.

15. **E**) $|A-\lambda I|=\begin{vmatrix}1-\lambda & 2 & 1\\ 3 & 4-\lambda & 2\\ 4 & 8 & 2-\lambda\end{vmatrix}$

$=(1-\lambda)(2-\lambda)(4-\lambda)+24+16-4(4-\lambda)-6(2-\lambda)-16(1-\lambda)$

The coefficient of term λ^2 is $\lambda_1+\lambda_2+\lambda_3$=7. No correct answer provided so the correct answer is E.

16. **A**) $(A-5I)V=\begin{bmatrix}6-5 & -1\\ 2 & 3-5\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0\Rightarrow\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}$. The correct answer is A.

17. **B**) Suppose $V\cdot U=1\cdot 2+2\cdot 0+3\cdot 4=14$. The correct answer is B.

18. **C**) Suppose $V\times U=\left(2\cdot 2-4\cdot 0 \quad 4\cdot 3-1\cdot 2 \quad 1\cdot 0-2\cdot 3\right)=\left(4 \quad 10 \quad -6\right)$. The correct answer is C.

19. **D**) $3a^2+2=14\Rightarrow a^2=4\Rightarrow a=\pm 2$. The correct answer is D.

20. **A**) $\cos\theta = \frac{U\cdot V}{|U||V|} = \frac{6+3-2}{\sqrt{14}\sqrt{14}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$. The correct answer is A.

21. **B**) $V = \begin{pmatrix}-1 & 4 & 2\end{pmatrix}$ onto $U = \begin{pmatrix}1 & 0 & 3\end{pmatrix} = \frac{U\cdot V}{|U|^2}U = \frac{-1+0+6}{\sqrt{10}\sqrt{10}}\begin{pmatrix}1 & 0 & 3\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & 0 & \frac{3}{2}\end{pmatrix}$.

The correct answer is B.

22. **C**) $V = \begin{pmatrix}1 & 2\end{pmatrix}$ onto $U = \begin{pmatrix}-1 & 1\end{pmatrix} = \frac{U\cdot V}{|U|} = \frac{-1+2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$.

The correct answer is C.

23. **D**) $A^0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. The correct answer is C.

24. **E**) $\det(A) = \begin{vmatrix}2 & 3\\ 3 & 2\end{vmatrix} - 2\begin{vmatrix}0 & 3\\ 1 & 2\end{vmatrix} + \begin{vmatrix}0 & 2\\ 1 & 3\end{vmatrix} = -5+6-2 = -1$. No correct answer provided. The correct answer is E.

~~25.~~ **Thrown out** ~~**D)** Let $f(x) = x^2 - 5x + 2$ and~~

~~$f(A) = \begin{pmatrix}1 & 0\\ 2 & 5\end{pmatrix}\begin{pmatrix}1 & 0\\ 2 & 5\end{pmatrix} - 5\begin{pmatrix}1 & 0\\ 2 & 5\end{pmatrix} + 2I = \begin{pmatrix}1 & 0\\ 2 & 5\end{pmatrix}\begin{pmatrix}-4 & 0\\ 2 & 0\end{pmatrix} + 2\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}-2 & 0\\ 2 & 2\end{pmatrix}$, The correct answer is D.~~

~~26.~~ **Thrown out** ~~**C)**~~

~~$A^2 - 2A + 5 = 0 \Rightarrow A(A-2I) + 5 = 0 \Rightarrow A(2I - A) = 5 \Rightarrow A\frac{(2I-A)}{5} = I \Rightarrow A^{-1} = \frac{(2I-A)}{5}$. The correct answer is C.~~

27. **B**) $V\cdot U = 2(3+y) + y - x + 4(x - \frac{3}{2}) = 3y + 3x = 0$. The correct answer is B.

28. **A**) $\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ 2 & 2 & 1\\ 4 & 3 & 1\end{vmatrix} = \frac{1}{2}(2+6-8-3) = -\frac{3}{2}$. The correct answer is A.

29. **A**) $\begin{vmatrix}1 & y & 1\\ x & 2 & 1\\ -1 & 2 & 1\end{vmatrix} = 2 + 2x - y + 2 - 2 - xy = 0 \Rightarrow (x+1)(y-2) = 0$. The correct answer is A.

30. **B)** $\left|AA^{-1}\right| = 1 \Rightarrow \left|A^{-1}\right| = \frac{1}{4}$, Therefore, $\left|4A^{-1}\right| = 4^3\left|A^{-1}\right| = 4^2$. The correct answer is B.